

**DIRECT DETERMINATIONS OF THE REDSHIFT
BEHAVIOR OF THE PRESSURE, ENERGY DENSITY, AND
EQUATION OF STATE OF THE DARK ENERGY AND THE
ACCELERATION OF THE UNIVERSE**

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One of the goals of current cosmological studies is the determination of the expansion and acceleration rates of the universe as functions of redshift, and the determination of the properties of the dark energy that can explain these observations. Here the expansion and acceleration rates are determined directly from the data, without the need for the specification of a theory of gravity, and without adopting an a priori parameterization of the form or redshift evolution of the dark energy. We use the latest set of distances to SN standard candles from Riess et al. (2004), supplemented by data on radio galaxy standard ruler sizes, as described by Daly & Djorgovski (2003, 2004). We find that the universe transitions from acceleration to deceleration at a redshift of $z_T \approx 0.4$, with the present value of $q_0 = -0.35 \pm 0.15$. The standard “concordance model” with $\Omega_0 = 0.3$ and $\Lambda = 0.7$ provides a reasonably good fit to the dimensionless expansion rate as a function of redshift, though it fits the dimensionless acceleration rate as a function of redshift less well. The expansion and acceleration rates are then combined with a theory of gravity to determine the pressure, energy density, and equation of state of the dark energy as functions of redshift. Adopting General Relativity as the correct theory of gravity, the redshift trends for the pressure, energy density, and equation of state of the dark energy out to $z \sim 1$ are determined, and are found to be generally consistent with the concordance model; they have zero redshift values of $p_0 = -0.6 \pm 0.15$, $f_0 = 0.62 \pm 0.05$, and $w_0 = -0.9 \pm 0.1$.

1. Introduction

One way to determine the expansion and acceleration rates of the universe as functions of redshift is through studies of the coordinate distance to sources at different redshift. This can be accomplished with a variety of techniques including the use of supernovae and radio galaxies (e.g. Riess et al. 2004; Perlmutter et al. 1999; Riess et al. 1998; Daly 1994; Guerra & Daly 1998; Guerra, Daly, & Wan 2000). The techniques fall into two broad categories: the integral and the differential approaches.

The former, traditional approach involves the integration of a theoretically predicted expansion rate over redshift to obtain predicted coordinate distances to different redshifts; the difference between these predicted coordinate distances and the observed coordinate distances is then minimized to obtain the best fit model parameters. This approach usually requires the specification of a theory of gravity (generally taken to be General Relativity; GR) and a parameterization of the redshift evolution of the dark energy. Maor, Brustein, & Steinhardt (2001) and Barger & Marfatia (2001) discuss how difficult it is to extract the redshift behavior of the dark energy using this method. Some techniques have been developed to extract the redshift behavior of the dark energy using the integral method (e.g. Starobinsky 1998; Huterer & Turner 1999, 2001; Saini et al. 2000; Chiba & Nakamura 2000; Maor, Brustein, & Steinhardt 2001; Golaith et al. 2001; Wang & Garnavich 2001; Astier 2001; Gerke & Efstathiou 2002; Weller & Albrecht 2002; Padmanabhan & Choudhury 2002; Tegmark 2002; Huterer & Starkman 2003; Sahni et al. 2003; Alam et al. 2003; Wang & Freese 2004; Wang et al. 2004; Wang & Tegmark; Nessier & Perivolaropoulos 2004; Gong 2004; Zhu, Fujimoto, & He 2004; Elgaroy & Multamaki 2004; Huterer & Cooray 2004; Alam, Sahni, & Starobinsky 2004).

2. The Methodology

The differential approach has been investigated by Daly & Djorgovski (2003, 2004), and we offer a brief summary here. It is well known (e.g. Weinberg 1972; Peebles 1993; Peebles & Ratra 2003) that the dimensionless expansion rate $E(z)$ can be written as the derivative of the dimensionless coordinate distances $y(z)$; the expression is particularly simple when the space curvature term is equal to zero. In this case,

$$\left(\frac{\dot{a}}{a}\right) H_0^{-1} \equiv E(z) = (dy/dz)^{-1}, \quad (1)$$

where a is the cosmic scale factor, and Hubble's constant is $H_0 = (\dot{a}/a)$ evaluated at zero redshift. This representation follows directly from the Friedman-Robertson-Walker line element, and does not require the use of a theory of gravity. Similarly, in a spatially flat universe (as convincingly demonstrated by CMBR measurements, Spergel et al. 2003), it is shown in Daly & Djorgovski (2003) that the dimensionless deceleration parameter

$$-\left(\frac{\ddot{a}a}{\dot{a}^2}\right) \equiv q(z) = -[1 + (1+z)(dy/dz)^{-1} d^2y/dz^2] \quad (2)$$

also follows directly from the FRW line element, and is independent of any assumptions regarding the dark energy or a theory of gravity. Thus, measurements of the dimensionless coordinate distance to sources at different redshifts can be used to determine dy/dz and d^2y/dz^2 , which can then be used to determine $E(z)$ and $q(z)$, and these direct determinations are completely model-independent, as discussed by Daly & Djorgovski (2003).

In addition, if a theory of gravity is specified, the measurements of dy/dz and d^2y/dz^2 can be used to determine the pressure, energy density, and equation of state of the dark energy as functions of redshift (Daly & Djorgovski 2004); we assume the standard GR for this study. *These determinations are completely independent of any assumptions regarding the form or properties of the dark energy or its redshift evolution.* Thus, we can use the data to determine these functions directly, which provides an approach that is complementary to the standard one of assuming a physical model, and then fitting the parameters of the chosen function.

In a spatially flat, homogeneous, isotropic universe with non-relativistic matter and dark energy Einstein's equations are $(\ddot{a}/a) = -(4\pi G/3) (\rho_m + \rho_{DE} + 3P_{DE})$ and $(\dot{a}/a)^2 = (8\pi G/3) (\rho_m + \rho_{DE})$, where ρ_m is the mean mass-energy density of non-relativistic matter, ρ_{DE} is the mean mass-energy density of the dark energy, and P_{DE} is the pressure of the dark energy. Combining these equations, we find $(\ddot{a}/a) = -0.5[(\dot{a}/a)^2 + (8\pi G) P_{DE}]$.

Defining the critical density at the present epoch in the usual way, $\rho_{oc} = 3H_0^2/(8\pi G)$, it is easy to show that $p(z) \equiv (P_{DE}(z)/\rho_{oc}) = (E^2(z)/3) [2q(z) - 1]$. Combining this expression with eqs. (1) and (2) we obtain the pressure of the dark energy as a function of redshift in terms of first and second derivatives of the dimensionless coordinate distance y (Daly & Djorgovski 2004)

$$p(z) = -(dy/dz)^{-2} [1 + (2/3) (1+z) (dy/dz)^{-1} (d^2y/dz^2)] . \quad (3)$$

Thus, the pressure of the dark energy can be determined directly from measurements of the coordinate distance. In addition, this provides a direct measure of the cosmological constant for Friedmann-Lemaitre models since in these models $p = -\Omega_\Lambda$. If more than one new component is present, this pressure is the sum of the pressures of the new components.

Similarly, the energy density of the dark energy can be obtained directly from the data

$$f(z) \equiv \left(\frac{\rho_{DE}(z)}{\rho_{oc}} \right) = (dy/dz)^{-2} - \Omega_0(1+z)^3, \quad (4)$$

where $\Omega_0 = \rho_{om}/\rho_{oc}$ is the fractional contribution of non-relativistic matter to the total critical density at zero redshift, and it is assumed that this non-relativistic matter evolves as $(1+z)^3$. If more than one new component is present, then f includes the sum of the mean mass-energy densities of the new components.

The equation of state $w(z)$ is defined to be the ratio of the pressure of the dark energy to its energy-density $w(z) \equiv P_{DE}(z)/\rho_{DE}(z)$. As shown by Daly & Djorgovski (2004), the equation of state is

$$w(z) = - \frac{[1 + (2/3)(1+z)(dy/dz)^{-1}(d^2y/dz^2)]}{[1 - (dy/dz)^2 \Omega_0(1+z)^3]}. \quad (5)$$

Here, w is the equation of state of the dark energy; if more than one new component contributes to the dark energy, w is the ratio of the sum of the total pressures of the new components to their total mean mass-energy densities.

3. Results and Conclusions

The results presented here follow those presented by Daly & Djorgovski (2003, 2004), where more details can be found. The data used here includes 20 radio galaxies (RG) compiled by Guerra, Daly, & Wan (2000) and the “gold” supernova (SN) sample compiled by Riess et al. (2004). We note that in the redshift interval where the two sets of coordinate distances (RG and SN) overlap, the agreement is excellent, suggesting that neither one is affected by some significant bias, and allowing us to combine them for this study.

Measurements of luminosity distances and angular size distances are easily converted to coordinate distances, $y(z)$. Using some robust numerical differentiation method, these can be used to determine the first and second derivatives as functions of redshift, which can be combined to determine the

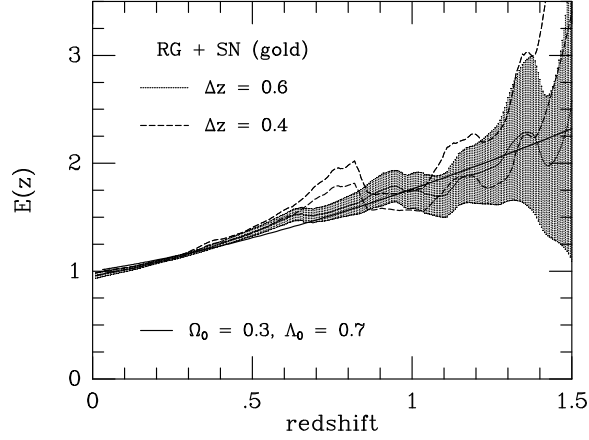


Figure 1. The derived values of the dimensionless expansion rate $E(z) \equiv (\dot{a}/a)H_0^{-1} = (dy/dz)^{-1}$ obtained with window functions of width $\Delta z = 0.4$ and their 1σ error bars (dashed lines) and 0.6 (dotted line and hatched error range). At zero redshift, the value of E is $E_0 = 0.97 \pm 0.03$. The value of $E(z)$ predicted in a spatially flat universe with a cosmological constant $\Lambda = 0.7$ and mean mass density $\Omega_0 = 0.3$ is also shown.

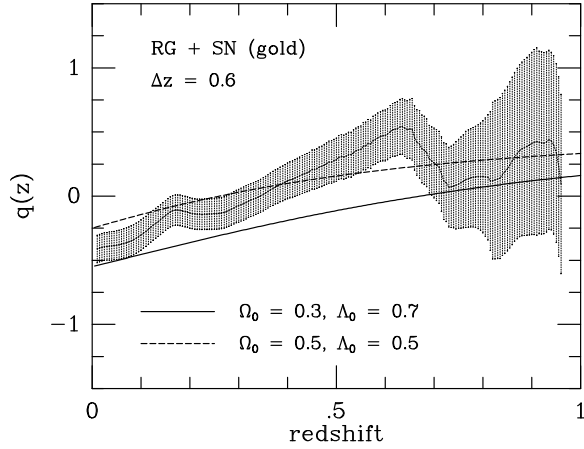


Figure 2. The derived values of deceleration parameter $q(z)$ and their 1σ error bars obtained with window function of width $\Delta z = 0.6$ applied to the radio galaxy and gold supernovae samples. The universe transitions from acceleration to deceleration at a redshift $z_T \approx 0.4$, with an uncertainty difficult to quantify due to large fluctuations at $z > 0.5$, caused by the sparseness of the data at higher redshifts. The present value is $q_0 = -0.35 \pm 0.15$. Solid and dashed lines show the expected dependence in the standard Friedmann-Lemaître models with zero curvature, for two pairs of values of Ω_0 and Λ_0 .

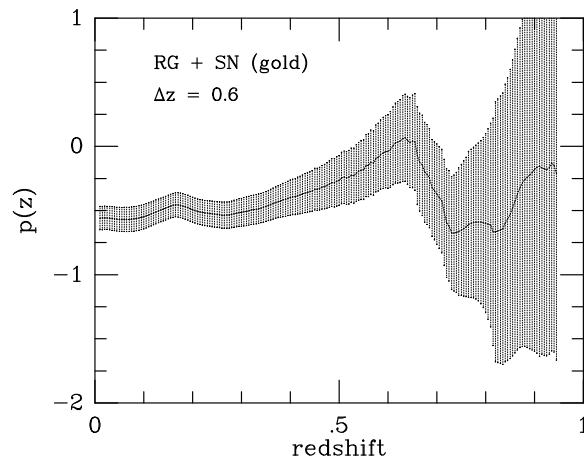


Figure 3. The derived values of dark energy pressure $p(z)$, obtained with window function of width $\Delta z = 0.6$. This derivation of $p(z)$ requires a choice of theory of gravity, and GR has been adopted here. The present value is $p_0 = -0.6 \pm 0.15$. Note that in the standard Friedmann-Lemaître models, $p_0 = -\Lambda_0$, and thus we have a direct measurement of the value of the cosmological constant, which is also fully consistent with other modern measurements.

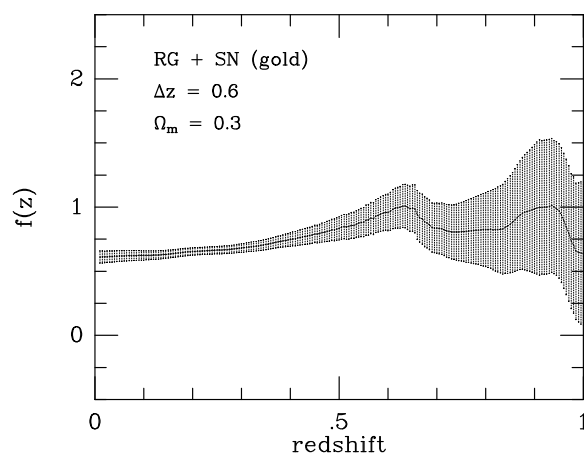


Figure 4. The derived values of the dark energy density fraction $f(z)$, obtained with window function of width $\Delta z = 0.6$. This derivation of $f(z)$ requires of theory of gravity and the value of Ω_0 for the nonrelativistic matter; GR has been adopted here, and $\Omega_0 = 0.3$ is assumed. The present value is 0.62 ± 0.05 , and the trend is consistent with $f(z) = \text{const.}$ out to $z \approx 1$.

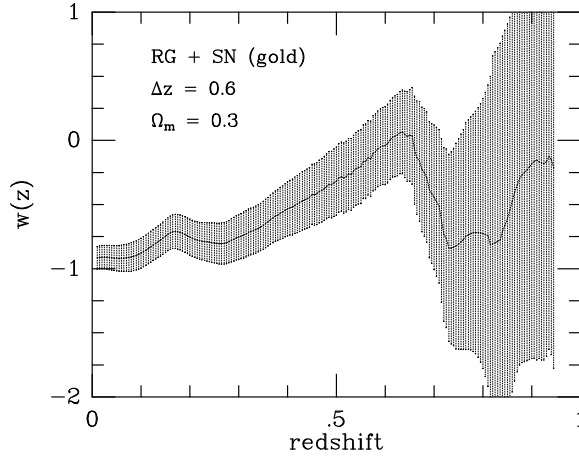


Figure 5. The derived values of the dark energy equation of state parameter $w(z)$, obtained with window function of width $\Delta z = 0.6$. This derivation of $w(z)$ requires of theory of gravity and the value of Ω_0 ; GR has been adopted here, and $\Omega_0 = 0.3$ is assumed. The present value is $w_0 = -0.9 \pm 0.1$, consistent with cosmological constant models.

dimensionless expansion and acceleration rates of the universe as functions of redshift, and the pressure, energy density, and equation of state of the dark energy as functions of redshift, as described above.

We see that the universe transitions from acceleration to deceleration at a redshift of about 0.4 (consistent with determinations by Daly & Djorgovski 2003, Riess et al. 2004, and Alam, Sahni, & Starobinsky 2004); and our determination only depends upon the assumption that the universe is homogeneous, isotropic, and spatially flat. Assuming GR, we solve for the pressure, energy density, and equation of state of the dark energy. Each is generally consistent with remaining constant to a redshift of about 0.5 and possibly beyond, but determining their behavior at higher redshifts is severely limited by the available data.

As more and better data become available, this methodology can be used to determine the evolution of the dark energy properties and the observed kinematics of the universe with an increasing precision and confidence.

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